Math 247: Confidence Intervals for Two Proportions (Section 7.5)

Let's consider Dr. Saraux's penguins again. Here's the situation:

"Does banding penguins in order to study and track them actually cause harm to the penguins? Claire Saraux, a wildlife researcher, conducted an experiment to answer this question. In 1998, she tagged 200 king penguins at Possession Island in Antarctica. She fitted half the birds with steel bands, and the other half with internal electronic tags. After 10 years of monitoring, Saraux found that just 20 of banded birds had survived the decade, compared to 36 of electronically tagged ones."

What was the difference in survival rates (proportions) between the banded and unbanded penguins in the samples? Fill in the proper notation.

$$\text{Diff} = \hat{p}_1 - \hat{p}_2 = .20 - .36 = -.16$$

Parking Lot

Banded	Unbanded
$x_1 = 20$	$x_2 = 36$
$n_1 = 100$	$n_2 = 100$
$\hat{p}_1 = .20$	$\hat{p}_2 = .36$

So the question is whether this **difference** in the samples is just due to chance (sampling variability) or whether it shows that banding CAUSES (remember, this is a controlled experiment!) a decrease in survival rates. We explored this question via hypothesis testing, but another way to look at it is to calculate what the **difference** in population survival proportions is, based on the difference in sample survival proportions.

We can use a Confidence Interval to do this:

Format: **Estimated Difference** $\pm$ **Margin of Error**

$$(\hat{p}_1 - \hat{p}_2) \pm z^* \mathrm{SE}_{EST} \qquad \mathrm{SE}_{\mathrm{ESTIMATE}} = \sqrt{\frac{\hat{p}_1(1-\hat{p}_1)}{n_1} + \frac{\hat{p}_2(1-\hat{p}_2)}{n_2}}$$

(note: this is the Unpooled Standard Error…we use this since we aren't assuming the populaton proportions are equal as we do with hypothesis testing.)

By hand, find the 95% confidence interval for the difference in survival proportions in the POPULATIONS between the banded and unbanded penguins. Interpret the CI.

By hand: 95% confidence

$\Rightarrow z^* = 1.96$

$$SE_{est} = \sqrt{\frac{.20(.80)}{100} + \frac{.36(.64)}{100}} = .062$$

$$\hat{p}_1 - \hat{p}_2 \pm z^* \cdot SE_{est}$$

$$-.16 \pm 1.96(.062)$$

$$-.16 \pm \underbrace{.122}$$

$\rightarrow (-.16 - .122, -.16 + .122) = (-.282, -.038)$

This is a pretty large Margin of Error. The sample size is pretty small (for proportions)

Reminder:
$5 - 9 = -4$
smaller − bigger = NEG

Interpret. CI (95%): (−.282, −.038)

- The CI gives us a range of values showing how big the DIFFERENCE could be in survival rates for ALL penguins, if they were banded vs. not banded
- The DIFFERENCE is $p_{banded} - p_{no\ band}$ = NEG
- The CI values being negative tells us the banded penguins have a lower survival rate than the unbanded.

Thus, we are 95% confident that banding reduces* survival rate anywhere from 3.8% to 28.2%

* Remember that since this is an experiment we can conclude cause-and-effect!

How does this relate to the hypothesis test we did previously?

H0: $p_{band} - p_{no\ band} = 0$ ZERO difference in survival rates

Ha: $p_{band} - p_{no\ band} \neq 0$ There is a difference in survival rates.

Notice that the NULL is not in the CI → $\boxed{\text{ZERO}}$ is not a number in the interval (−.282, −.038)

So the CI is evidence against the null, so we would reject the null, at the .05 level of significance.

Now let's use StatCrunch to do the busy work for us and focus on the conclusion.

Use the same steps as you would for performing a hypothesis test, but check the "Confidence Interval" box.

→ STAT → PROPORTION → TWO SAMPLE → fill in values from the Parking Lot

What you should see from StatCrunch:

Two sample proportion summary confidence interval:

p_1 : proportion of successes for population 1

p_2 : proportion of successes for population 2

$p_1 - p_2$: Difference in proportions

95% confidence interval results:

C.I.

Difference	Count1	Total1	Count2	Total2	Sample Diff.	Std. Err.	L. Limit	U. Limit
$p_1 - p_2$	20	100	36	100	-0.16	0.062	-0.282	-0.0375

Now, let's look further into how the confidence interval relates to the null hypothesis, which states that there is ZERO DIFFERENCE between survival rates for banded vs. unbanded penguins, if we were to have half of ALL penguins banded and the other half unbanded. (i.e. in the population, not just the sample)

H0: $p_{band} - p_{no\ band} = 0$ ZERO difference – banding does NOT affect long term survival. {Banding does NOT affect long term survival.}

Scenario 1: Original data (20 banded survived, 36 unbanded survived for 10 years)
95% confidence interval for the **difference** in survival rates for the original data. Graph the interval below.

95% confidence interval results:

Difference	Count1	Total1	Count2	Total2	Sample Diff.	Std. Err.	L. Limit	U. Limit
$p_1 - p_2$	20	100	36	100	-0.16	0.062	-0.282	-0.0375

Previous page: Banding REDUCES survival

Scenario 2: Suppose 30 of the banded penguins survived and 36 of the unbanded penguins survived. Use StatCrunch to find the 95% confidence interval for the difference in survival rate.

L. Limit	U. Limit
-0.190	0.0701

Reflect on the difference in the sample proportions. Does it seem to be significant?

$\hat{p}_1 = \frac{30}{100} = .3$ Banded $\quad \hat{p}_2 = \frac{36}{100} = .36$ Unbanded

$\hat{p}_1 - \hat{p}_2 = .30 - .36 = -.06$

6% difference in the SAMPLES!
(Not that big of a difference ...)

Reflect on the CI.

CI: (−.190, +.0701)
(NEG, POS)

Neg ⇒ banding reduces survival in Pop
POS ⇒ banding INCREASES " " "
ZERO difference is a possibility!

Scenario 3: Now suppose 45 banded penguins survived vs. 36 unbanded.

L. Limit	U. Limit
-0.0455	0.225

Reflect on the difference in the sample proportions, then reflect on the CI.

Banded $\hat{p}_1 = \frac{45}{100} = .45$ $\quad$ Unbanded $\hat{p}_2 = \frac{36}{100} = .36$

$\hat{p}_1 - \hat{p}_2 = +.09$! What?!

CI: (−.046, +.225)
Again, the CI tells us nothing interesting – banding might hurt (−), might help (+), might make ZERO diff!

Scenario 4: Finally, suppose that 50 banded penguins survived vs. 36 unbanded.

L. Limit	U. Limit
+ 0.004	+ 0.276

Reflect on the difference in the sample proportions, then reflect on the CI.

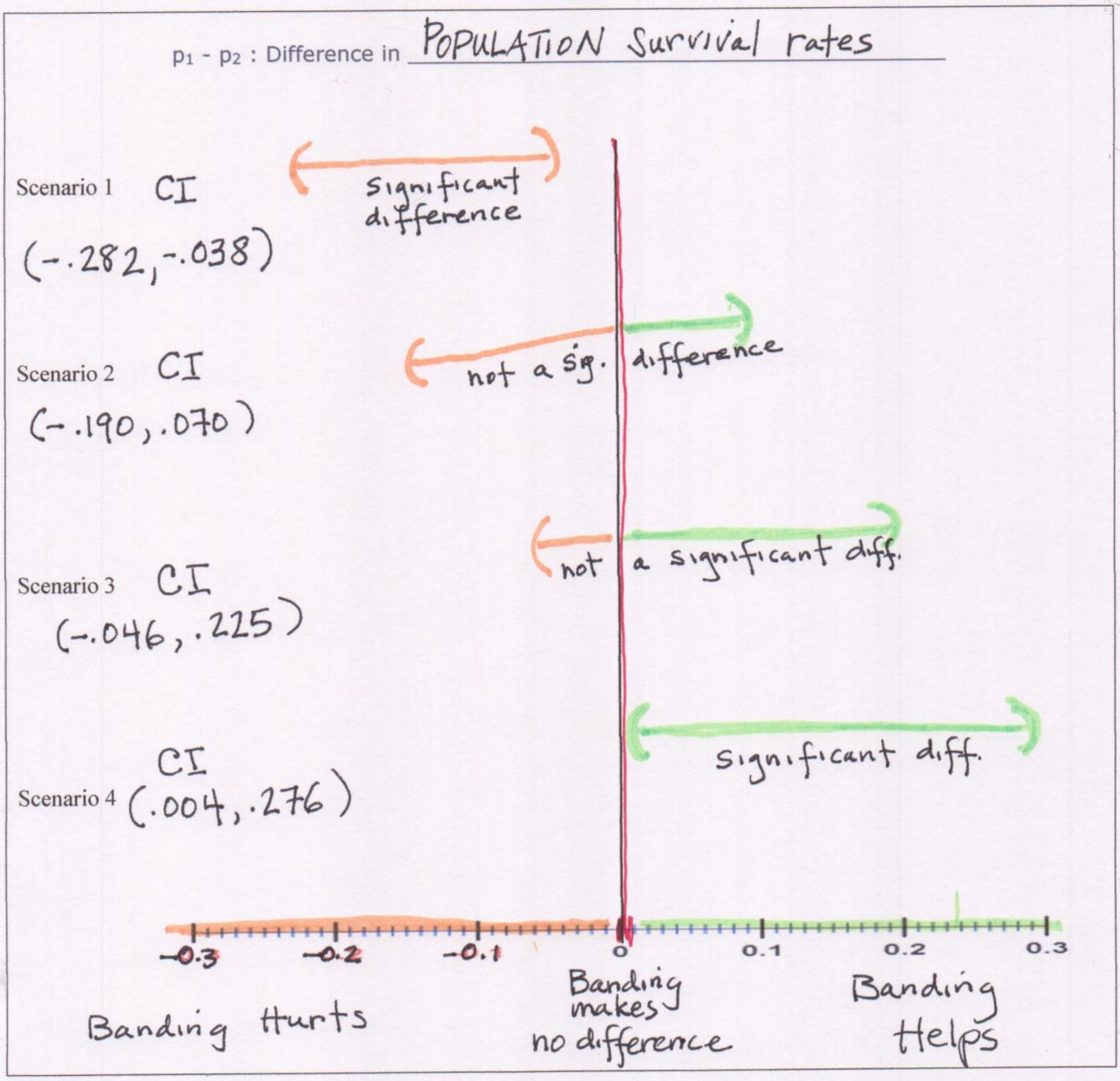
$\hat{p}_1 = \frac{50}{100} = .5$ Banded $\quad \hat{p}_2 = \frac{36}{100} = .36$ Unbanded

$\hat{p}_1 - \hat{p}_2 = .14$

Pretty big difference
MAY be significant

CI: (.004, .276)
Banding might INCREASE survival rates anywhere from .4% to 27.6%

Let's look at how this plays out in a graph. Graph the CI for each scenario.



Summary: When comparing the results from TWO SAMPLES, if the confidence interval "captures" zero then the data tells us it's <u>possible</u> there is ZERO difference between the population proportions.

- If the interval **captures zero,** we conclude there is **NO <u>significant</u> difference.**

  Null is in the CI

- If the interval **does not capture zero,** then we conclude there **IS a significant difference.**

  Null is <u>NOT</u> in the CI

  Evidence <u>AGAINST</u> the null