Math 247: Confidence Intervals for a Population Mean (Section 9.3)

<u>The Central Limit Theorem</u> for Means (Section 9.2) states that if a sample is selected such that the three conditions are met then the <u>distribution of sample means</u> follows an approximately Normal distribution, with standard error based on the population standard deviation.

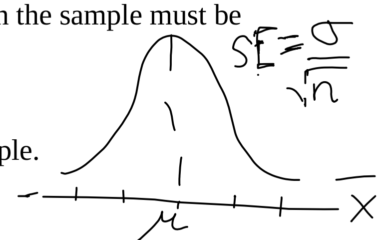
z-scores

Conditions that must be met **for Central Limit Theorem to apply** (page 416):

1. **Random and Independent.** The sample must be random and the observations in the sample must be independent from one another.
2. **~~Large~~ Sample.** Either the underlying (target) <u>population</u> distribution is Normal OR the sample size is large ($n \geq 25$)
3. **~~Large~~ Population.** The population must be at least 10 times the size of the sample.

We're going to ignore #3

$SE = \frac{\sigma}{\sqrt{n}}$



Major Problem: We almost never will know the <u>population</u> standard deviation! We can use the <u>sample</u> standard deviation as an estimate **<u>but that introduces more variability</u>**.

To the rescue! A new distribution curve (probability density function), **The Student's t-distribution**.

Features:

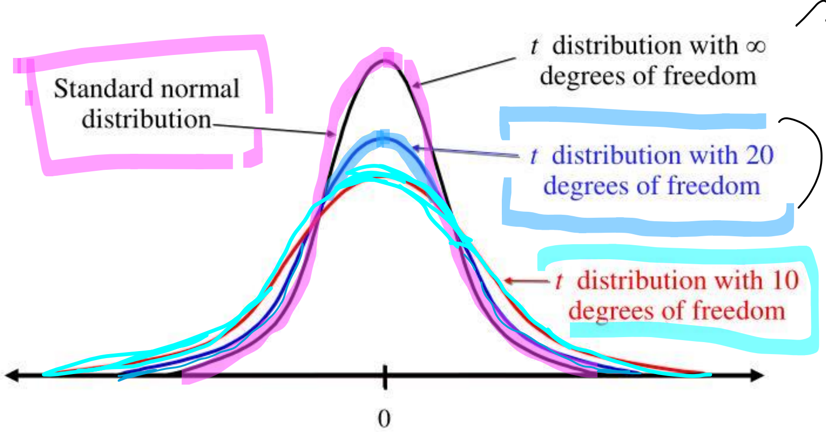
- The curve is *<u>bell-shaped</u>* like the normal distribution
- The curve is *<u>wider than the normal curve</u> (resulting in fatter tails, hence larger P-values)*
- The t-distribution has *<u>different curves</u>* based on sample size
- The sample size ~~is taken into~~ account by the "degrees of freedom" = df or

  Formula: $df = n - 1$

Smaller sample ⇒ Fewer degrees of freedom ⇒ wider curve ⇒ larger P-value, larger Margin of Error!

t Distribution – for all research using means

The t-distribution is used when n is **small** and σ is **unknown**.



medium-sized sample

small sample

So it's easier to find statistical significance (small P-value) when you have a larger sample, which makes sense!

last page of notes

Example: Use the Student's t-Distribution table or StatCrunch to find the following t* values.

Sample size is 30 df = 30 - 1 = 29	**Sample size is 101** df = 101 - 1 = 100
Confidence level: 90% t* = 1.699	Confidence level: 90% t* = 1.660
Confidence level: 95% t* = 2.045	Confidence level: 95% t* = 1.984
Confidence level: 99% t* = 2.756	Confidence level: 99% t* = 2.626

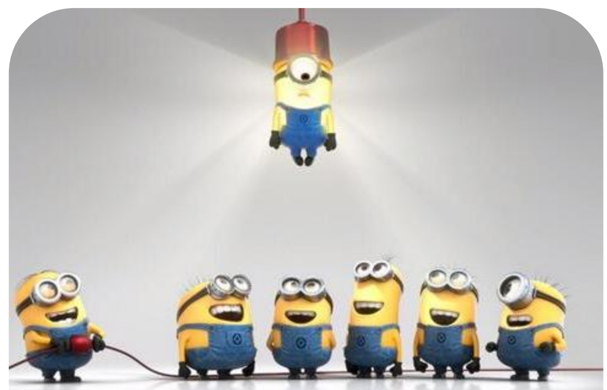
Constructing a Confidence Interval for a Population Mean:

Formula: $\bar{x} \pm t^* SE_{t-dist}$ $\qquad SE_{t-dist} = \dfrac{s}{\sqrt{n}}$

$\boxed{CI: \bar{x} \pm t^* \cdot \frac{s}{\sqrt{n}}}$

s = Sample S.D.

Example: **Lightbulb!** A company has developed a new type of energy-saving lightbulb, and wants to estimate the mean lifetime of the bulbs. A simple random sample of 30 bulbs had a mean lifetime of 1020 hours with a standard deviation of 45 hours.



Construct a 95% confidence interval for the mean lifetime of all bulbs manufactured by this new process. Do this by hand, using the formula. Then use StatCrunch (steps below) to find the confidence interval and interpret the results.

Find the CI by hand

Interpret: We are 95% confident the light bulbs last, on average, between 1003.2 hrs and 1036.8 hrs

Parking Lot

$n = 30$ bulbs

$\bar{x} = 1020$ hrs

$s = 45$ hrs

95% confidence

$t^* = 2.045$ $\qquad df = n - 1 = 29$

$$\bar{x} \pm t^* \frac{s}{\sqrt{n}}$$

$$1020 \pm \underbrace{2.045 \frac{45}{\sqrt{30}}}$$

$$\underbrace{1020}_{\text{Point Estimate}} \pm \underbrace{16.8}_{\text{Margin of Error}} \text{ hours}$$

Interval Format

(1020 − 16.8, 1020 + 16.8)

(1003.2, 1036.8) hrs

Find the confidence interval using StatCrunch:

StatCrunch steps for finding a CI of a single mean.

Steps:
1. If you have raw data, ~~enter~~ it into a column in a StatCrunch worksheet.
2. Click on **Stat**, then **T Stats**, then **One Sample**
3. Choose one of the following:
   - With Data: If you have entered the data into a column then click **With Data** and select the column
   - ~~With Summ~~ary: If you already have the sample mean and standard deviation, then click **With Summary** and enter the values.
4. Enter the confidence level in the **Confidence Level** field.
5. Click **Compute**.

One sample T summary confidence interval:

μ : Mean of population

95% confidence interval results:

Mean	Sample Mean	Std. Err.	DF	L. Limit	U. Limit
μ	1020	8.2158384	29	1003.1967	1036.8033

CI: (1003.2, 1036.8) hrs

there's 5% chance we got a weird sample.

Interpretation: We are 95% confident that the true average lifespan for these bulbs is between 1003.2 hrs and 1036.8 hrs.

Follow-up question:

(1) Does this result tell us how long any individual lightbulb will last? NO! The CI only tells us about averages (mean), not about individuals!!

(2) The company is going to advertise these bulbs as having an average lifespan of 1000 hours. However, they will get in legal trouble for false advertising if it was found that the average lifetime of these bulbs was actually significantly less than 1000 hours. Based on the confidence interval, should the company go ahead with its marketing plan?

YES, because the CI was over 1000 hours it's reasonable to believe these bulbs will last longer than 1000 hrs, ON AVERAGE.

5

Example: Nutrition. Kale is a type of cabbage that is known for its high mineral content. Suppose a lab made ~~6~~ 5 measurements of kale to determine the calcium content (in mg). Each measurement was of 200 grams of chopped, boiled kale with the following results:

175mg 184mg 204mg 191mg 218mg

enter into C1 in Statcrunch C1 "Calcium" name

Is this raw data or summarized data? Raw

If the lab wants to use this data to construct a confidence interval for how much calcium is contained in a serving kale, on average, what conditions have to be met for using the Central Limit Theorem for Means, using the t-Distribution?

1. Random sample? Assume
Independent observations? Assume

2. Large Sample OR small sample with normal population
$n = 5 < 25$ nope!
n=5 is a small sample!
$\boxed{\text{Assume}}$ the distribution of calcium amounts in kale is normal (symmetric)
There is another type of hypothesis test to determine this

Use StatCrunch to construct a 90% confidence interval for the mean mineral content from the data (outlier removed):

StatCrunch Confidence Interval: (178.3, 210.5) mg of calcium

Sketch the CI:

MoE 16.1 MoE 16.1

178.3 194.4 210.5

What is the margin of error?

$210.5 - 194.4 = \boxed{16.1}$

$\dfrac{210.5 - 178.3}{2} = \boxed{16.1}$

Interpretation:

We are 90% confident that the kale servings, on average, have between 178.3 and 210.5 mg of calcium.
The average calcium content for all such kale servings is between 178.3 mg and 210.5 mg.

StatCrunch result for reference:

90% confidence interval results:

Variable	Sample Mean	Std. Err.	DF	L. Limit	U. Limit
Calcium	194.4	7.5670338	4	178.26824	210.53176

175

A nutrition website claims that that, on average, a 200 gram serving of kale has a calcium content of $\boxed{175 \text{ mg.}}$ Does your work above support this claim, at the .10 level of significance? Explain.

No, our work suggests that this number is too low, because our CI values are all above 175.

Change the confidence level to 95% in StatCrunch and graph the confidence interval by hand

95% confidence interval results:

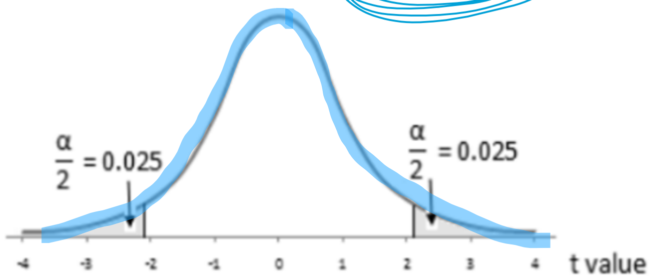
Variable	Sample Mean	Std. Err.	DF	L. Limit	U. Limit
Calcium	194.4	7.5670338	4	(173.39055	215.40945)

175

Does this CI support the claim that, on average, kale has $\boxed{175 \text{ mg}}$ of calcium per serving, at the .05 level of significance? Explain.

Since 175mg is "captured by" the CI, we certainly can't refute her claim – it's a distinct possibility!

Student's t-Distribution Table, for finding t* values



Confidence	90	95	95.45	99	99.73
alpha	0.1000	0.0500	0.0455	0.0100	0.0027
df					
1	6.314	12.706	13.968	63.657	235.784
2	2.920	4.303	4.527	9.925	19.206
3	2.353	3.182	3.307	5.841	9.219
4	2.132	2.776	2.869	4.604	6.620
5	2.015	2.571	2.649	4.032	5.507
6	1.943	2.447	2.517	3.707	4.904
7	1.895	2.365	2.429	3.499	4.530
8	1.860	2.306	2.366	3.355	4.277
9	1.833	2.262	2.320	3.250	4.094
10	1.812	2.228	2.284	3.169	3.957
11	1.796	2.201	2.255	3.106	3.850
12	1.782	2.179	2.231	3.055	3.764
13	1.771	2.160	2.212	3.012	3.694
14	1.761	2.145	2.195	2.977	3.636
15	1.753	2.131	2.181	2.947	3.586
16	1.746	2.120	2.169	2.921	3.544
17	1.740	2.110	2.158	2.898	3.507
18	1.734	2.101	2.149	2.878	3.475
19	1.729	2.093	2.140	2.861	3.447
20	1.725	2.086	2.133	2.845	3.422
21	1.721	2.080	2.126	2.831	3.400
22	1.717	2.074	2.120	2.819	3.380
23	1.714	2.069	2.115	2.807	3.361
24	1.711	2.064	2.110	2.797	3.345
25	1.708	2.060	2.105	2.787	3.330
26	1.706	2.056	2.101	2.779	3.316
27	1.703	2.052	2.097	2.771	3.303
28	1.701	2.048	2.093	2.763	3.291
29	1.699	2.045	2.090	2.756	3.280
30	1.697	2.042	2.087	2.750	3.270
40	1.684	2.021	2.064	2.704	3.199
50	1.676	2.009	2.051	2.678	3.157
60	1.671	2.000	2.043	2.660	3.130
70	1.667	1.994	2.036	2.648	3.111
80	1.664	1.990	2.032	2.639	3.096
90	1.662	1.987	2.028	2.632	3.085
100	1.660	1.984	2.025	2.626	3.077
1000	1.646	1.962	2.003	2.581	3.007
∞	1.645	1.960	2.000	2.576	3.000

#'s in the table are t*-values

$z^* = 1.960$ for 95% confidence