Math 247: Hypothesis Testing with Proportions (Sections 8.1, 8.2, 8.3)
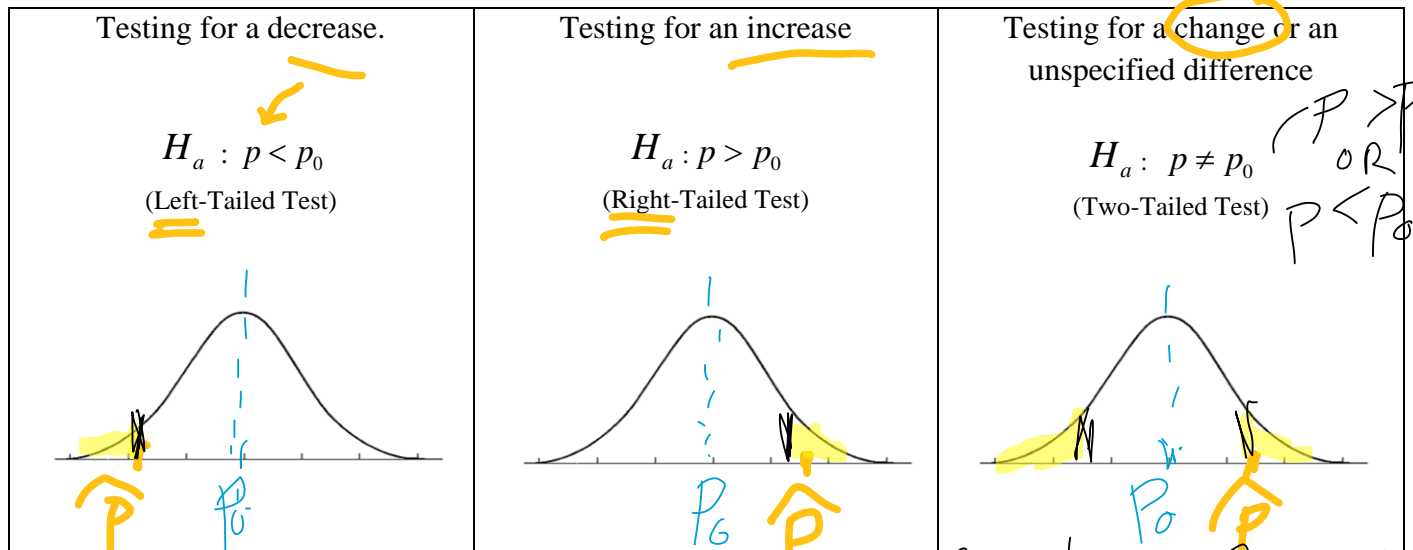
The Four Steps of Hypothesis Testing

Step 1: Hypothesize

Write down the hypotheses. Write them in **words** and in **math symbols**.

H₀ always → WORDS also!

Null hypothesis $H_0 : p = p_0$

Alternative hypothesis: Use one of the 3 options below:

Testing for a decrease.	Testing for an increase	Testing for a change or an unspecified difference
$H_a : p < p_0$	$H_a : p > p_0$	$H_a : p \neq p_0$
(Left-Tailed Test)	(Right-Tailed Test)	(Two-Tailed Test)

$p > p_0$ OR $p < p_0$

$\hat{p}$ p_0 p_0 $\hat{p}$ next section p_0 $\hat{p}$

p-value = 2·area in 1 tail

Step 2: Prepare.

- Set the level of significance, α. This is the amount of risk we're willing to take in rejecting a true null hypothesis (false positive). More on this later... The default is $\alpha = .05$ — *reasonable doubt*

- Choose the test: We'll be using the One Proportion z-Test.

  Check that the conditions for the test are satisfied. If you're doing the research, you would choose a sample size and gather data so that the assumptions will be satisfied.

  Conditions:
  1. Random, Independent? We have a random sample from the population and the observations are independent from one another.
     What to watch out for: We don't want the individuals in the sample to be associated in any way. For instance, we don't want members of the same family members included in the sample, or people in our survey discussing the question with one another before answering.

  2. Large Sample? The sample size is large enough to make the sampling distribution approximately normal.

     How to check this: The Expected Count must be at least 10 for each group:
     Success group: $E = np_0 \geq 10$?
     Failure group: $E = n(1 - p_0) \geq 10$

  3. Large Population? The population is at least 10 times bigger than the sample.

7.3

(handwritten: ~73)

Step 3: Compute to compare. Your goal here is to **find the Test Statistic and the p-value** which will tell you *how likely it is to get a value as or more extreme as the observed (known) value if the null hypothesis value (theoretical, not observed) is actually true.* In other words, what is the chance that the finding is due to just sampling variability?
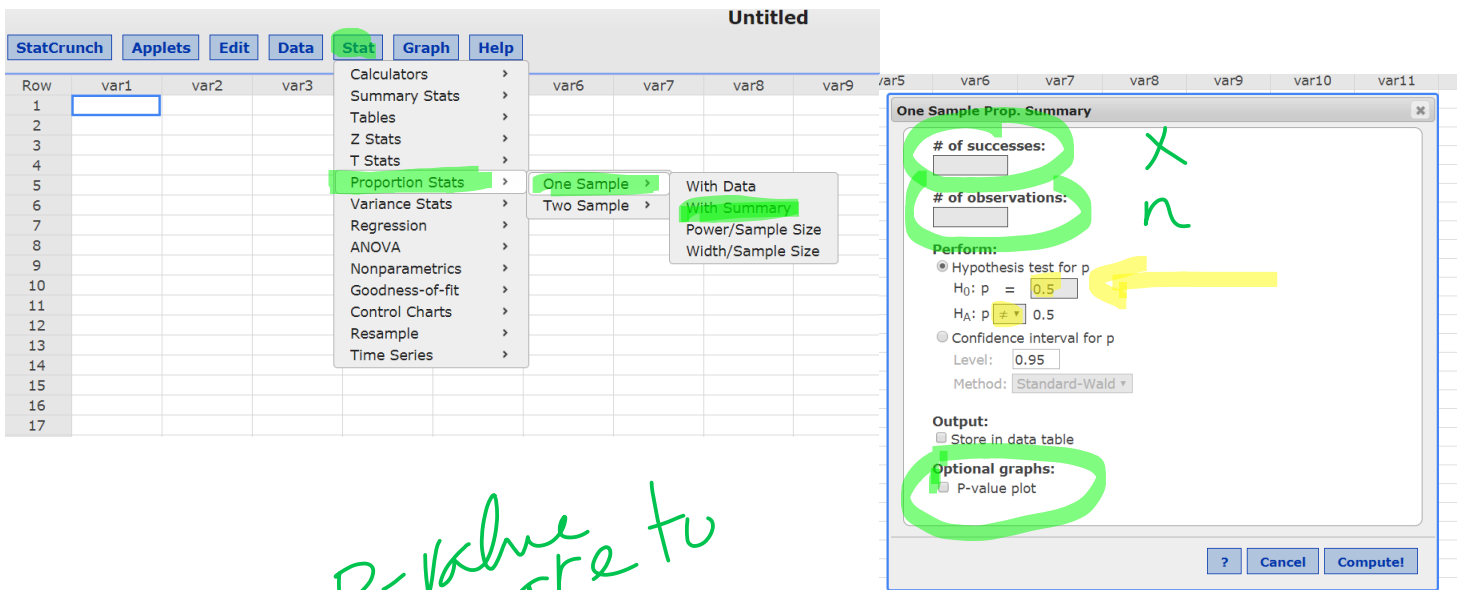
Test statistic: $z = \dfrac{\hat{p} - p}{SE}$

(handwritten: using Normal Dist Calculator)

P-value: We find the P-value using StatCrunch or by hand. In either case, for 1 Proportion z-Test we are using the Normal Approximation for the Sampling Distribution.

StatCrunch Instructions:

Open StatCrunch
Select "Stat" then "Proportion Stats" then "One Sample" then "With Summary"
Fill in the dialogue box. Check the "P-value plot" box.
Compute!
Copy the results table and the graph into your paper for the StatCrunch homework.

(handwritten: P-value to compare to α)

Step 4: Interpret.

Is the P-value smaller than alpha?
    Reject the null, accept the alternative
    The finding IS statistically significant. (State this in the context of the problem.)

Or is the P-value greater than alpha?
    Fail to reject the null...do NOT "accept the null"
    The finding is NOT statistically significant. (State this in the context of the problem.)

"

Example: Happiest Place on Earth. Have you heard someone say that Oprah Winfrey called SLO the "happiest place on Earth"? Happiness is the subject of many local and global studies including those that find associations (which are often reported incorrectly as causation) between happiness and various factors such as health, social engagement, income, and the like.

— no info about sample, assume pop value

In 2017, the Harris Poll found that 33% of Americans reported being "Very Happy". (Source: http://time.com/4871720/how-happy-are-americans/) Suppose you're curious about whether SLO residents have a higher level of happiness from the overall US level. You conduct a survey of 200 randomly chosen people in SLO and find that 78 of them say that they're very happy. Does this data give you enough evidence to conclude that SLO happiness ("very happy" rate) is significantly different from overall US happiness? Use $\alpha = .05$ as the significance level.

Perform all four steps of a hypothesis test, using StatCrunch for Step 3, "Compute". **Parking Lot**

Step 1: **Hypothesize — before any data!**

$H_0: P_{SLO} = .33$ The proportion of very happy people in SLO is the same as in U.S. overall

$H_a: P_{SLO} \neq .33$

The proportion is different (more or less)

$P = .33$
P_0
$n = 200$
$X = 78$
$X = $ number of successes "Very Happy"

$\hat{P} = \dfrac{78}{200} = .39$ sample

Step 2: **Prepare**

Set $\alpha = .05$
Choose test
One Proportion Z-Test

Conditions:

✓ 1. Random sample? Stated
Independent observations? Assume
Keep separate?

✓ 2. Large sample?
Success — very happy
failure — not very happy?

$E = n p_0 \geq 10$?
$200(.33) = 66 \geq 10$ ✓
$E = n(1-p_0) \geq 10$?
$= 200(1-.33) = 134 \geq 10$ ✓

✓ 3. Large pop?
Pop of SLO residents $\geq 10(200) = 2000$ people in SLO?
yes

Step 3: **Compute — Joali get a**

Do this step by hand then use StatCrunch to check your result.

$H_0: P = .33$ $H_a: P \neq .33$
Two-tailed

$n = 200$
$x = 78$

$Z = \dfrac{\hat{p} - P_0}{SE}$

$\hat{p} = \dfrac{78}{200} = .39$

$= \dfrac{.39 - .33}{.033}$

$SE = \sqrt{\dfrac{P_0(1-P_0)}{n}}$

$= 1.818$

$= \sqrt{\dfrac{.33(1-.33)}{200}}$

$= .033 \rightarrow$ tick marks

Null $P(\hat{p} \geq .39)$
$= P(z \geq 1.818)$
$= .0345$
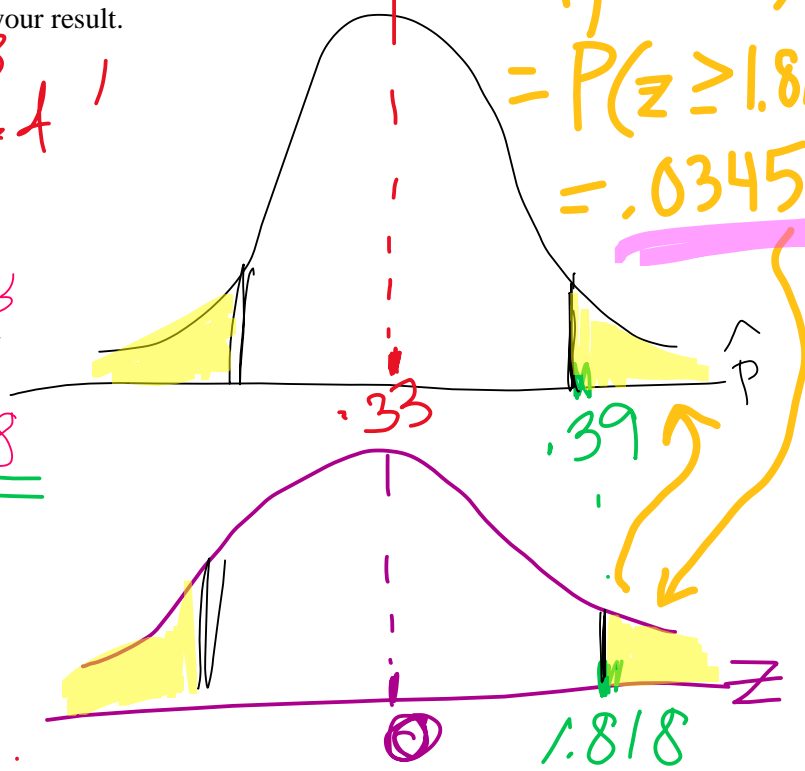
$P\text{-value} = 2 \cdot (.0345) = .069$

StatCrunch Results:

Proportion	Count	Total	Sample Prop.	Std. Err.	Z-Stat	P-value
p	78	200	0.39	0.03324906	1.8045623	0.0711

Standard Normal Distribution
Z-stat: 1.8046, P-value: 0.0711
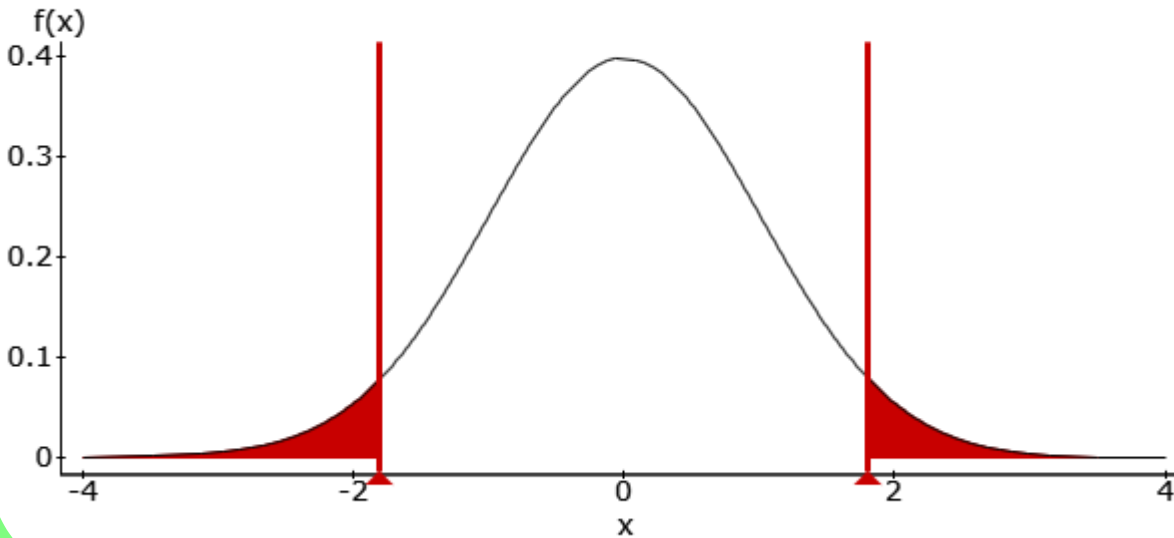
Step 4: __Interpret - make a decision__

P-value = .069 > .05 = α.
P-value is too big $\Rightarrow$ we aren't beyond "reasonable doubt" Fail to reject null. We do not have evidence that the proportion of very happy people in SLO is different than the proportion in the US as a whole

Follow-up questions:

1. Would the conclusion change if the significance level had been .10 instead of .05? Explain.

P-value = .069 < .10 = α reject the null!
Yes, we would now reject the null!
We would conclude that the proportion in SLO IS significantly different than in whole US!

2. Would the conclusion change if the Research Question were "Are people actually happier in SLO than in the rest of the US?" To investigate this, do the following:

Rewrite the hypotheses: (Remember the proportion of "very happy people" in the US was 33%)

$H_0 : p_{SLO} = .33$

$H_a : p_{SLO} > .33$

Is this a one-tailed or two-tailed test?

What would the P-value be in this situation? (Look at the previous work)

P-value $= \frac{1}{2}$(P-value of Two-Tailed Test!)
$= \frac{1}{2}(.069) = .0345$

What would the conclusion be based on the P-value?

P-value = .0345 < .05 = α
Reject the null — people are significantly happier in SLO!