Math 247: Hypothesis Testing for One Mean (Section 9.4)

Four Step Process

Step 1: Hypothesize

State hypotheses using math **symbols**. Then describe what those symbols are saying, in **words**.

$H_0 : \mu = \mu_0$ Fixed value

$H_a : \mu < \mu_0$ (Left-Tailed Test) or $\mu > \mu_0$ (Right-Tailed Test) or $\mu \neq \mu_0$ (Two-Tailed Test)

Step 2: Prepare

Choose and **state** which test you're using and choose the Significance Level, α (alpha)

Check Conditions (make assumptions) FOR THAT TEST!

For the t-Test for One Mean, we have to meet these conditions:

1. **Random** Sample and **Independent** Observations?
2. **Large Sample or Normal Population**? What to check:
   Either the sample is large ($n \geq 25$)
   or, if sample is small ($n < 25$), there must be evidence that the underlying population is approximately **normal**.

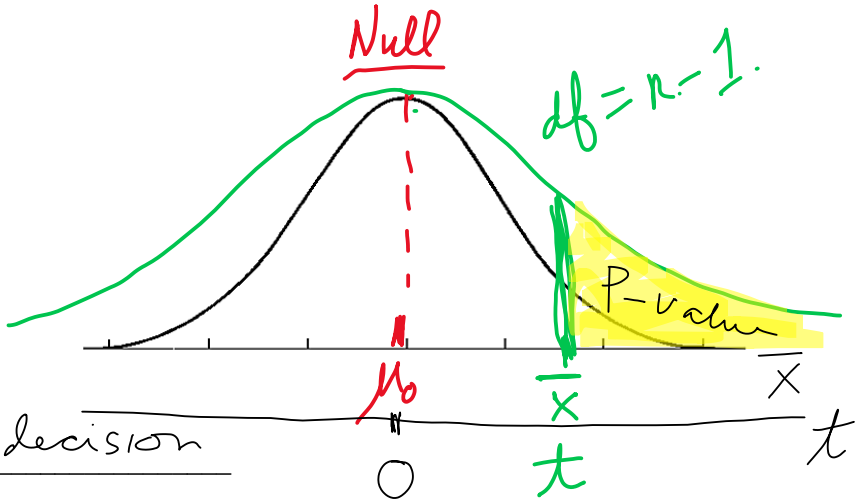
from the CLT

Step 3: Compute - get P-value

Find the Test Statistic by hand, but then we'll use StatCrunch to find the P-value. Be able to illustrate all of this on a t-distribution curve.

$$t = \frac{\bar{x} - \mu}{SE} \qquad SE = \frac{s}{\sqrt{n}}$$

$$df = n - 1$$



Step 4: Interpret - Make a decision

Include both of the following:

✓ (a) Compare the p-value to the level of significance and state whether you will reject or not reject the null hypothesis

✓ (b) Interpret the result in the context of the problem. Your interpretation should include whether or not your result is statistically significant.

Example: Nutrition (again). Kale is a type of cabbage commonly found in salad and used in cooking in many parts of the world. Measurements were made of the calcium content (in mg) of kale (200 grams of chopped, boiled kale) with the following results:

175mg 184mg 204mg 191mg 218mg

A nutrition website claims that that, on average, kale has a calcium content of 175 mg per 200 gram serving. Test to see whether the calcium content of kale is more than the website claims. **Use a significance level of .05**. Use StatCrunch to find the P-value but illustrate the StatCrunch results on a t-distribution curve.

1. $H_0: \mu = 175$ Mean calcium content of ALL such servings of kale is 175mg

$H_a: \mu > 175$ Mean calcium is higher than 175mg

Parking Lot: $\mu_0 = 175$mg

This is a One (Right) - Tailed Test

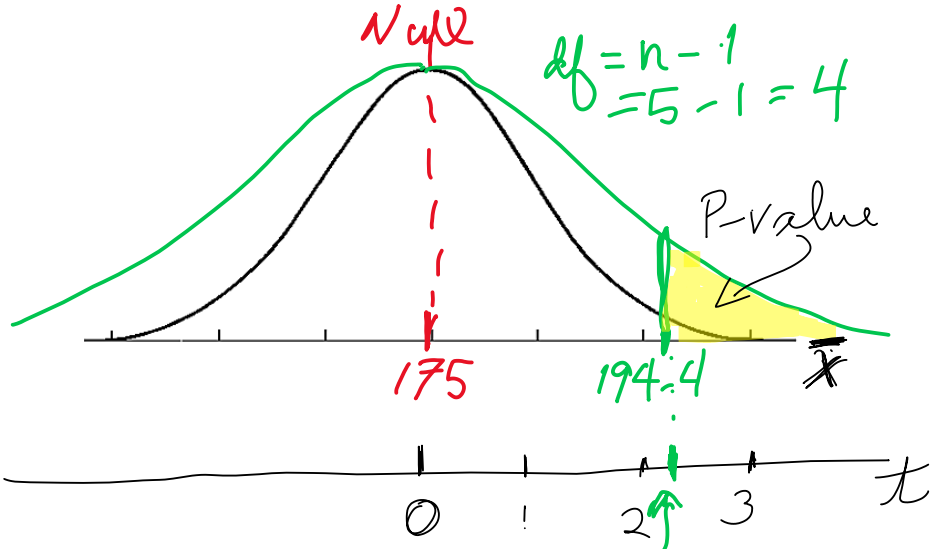
2. Choose test: t-Test for One Mean (One sample!)
   - $\alpha = .05$ level of significance
   - Check conditions
     1. Random sample? Assume Independent observations? Assume
     2. Large sample OR Normal Pop of calcium content
        $n = 5 < 25$ — small sample → Test or assume Pop is normal

3. Compute (StatCrunch)

$SE = \frac{s}{\sqrt{n}} = 7.567$

$t = \frac{\bar{x} - \mu_0}{SE} = \frac{194.4 - 175}{7.567}$

$t = 2.563$



Hypothesis test results:

Variable	Sample Mean	Std. Err.	DF	T-Stat	P-value
calcium	194.4	7.5670338	4	2.5637523	0.0312

4. Interpret

Reject H_0, accept H_a

P-value = .031 < .05 = α

(statistical)

The mean calcium content is significantly higher than 175 mg, based on this data.

Section 9.3

5. The 95% CI for calcium in a serving of kale was (173.4, 215.4) mg.

So, is it possible the website gave the correct value (average calcium content of a serving of kale is 175mg)?

Yes, because 175 mg is IN this CI!

$\mu = 175$ mg is a possible mean value for calcium content = don't reject null because the null is a possible value!

But the hypothesis test led us to conclude what about the average calcium content of kale?

We concluded that 175 mg is too low, the real value is higher than this! ?

Why is there a discrepancy in these two answers? What would the P-value be if we used a Two-Tailed Test and how would that change the conclusion?

> We're comparing the results of a One-Tailed Test to a CI - not an accurate comparison

Two-Tailed Test P-value = 2 · One-Tailed P-value

= 2 (.031)

= .062 > .05 ✓ Fail to reject H_0!

Summary: The best practice is to use a Two-Tailed Hypothesis Test, to be able to relate the confidence interval to the results of the hypothesis test.