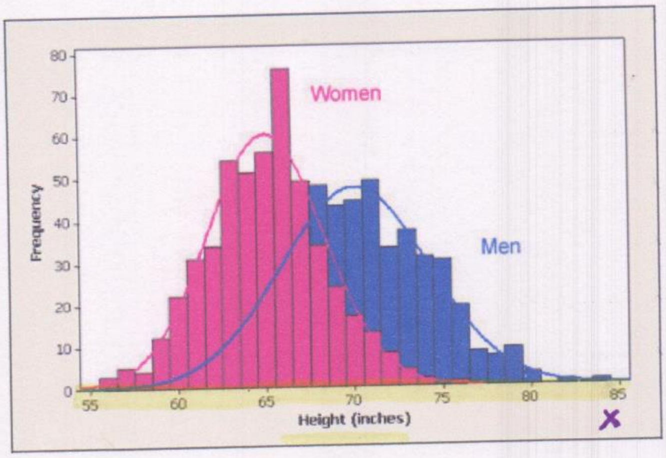
Continuation of 3.2: Z-scores, Empirical Rule

Math 247: Continuous Random Variables: The Normal Distribution (Section 6.2)

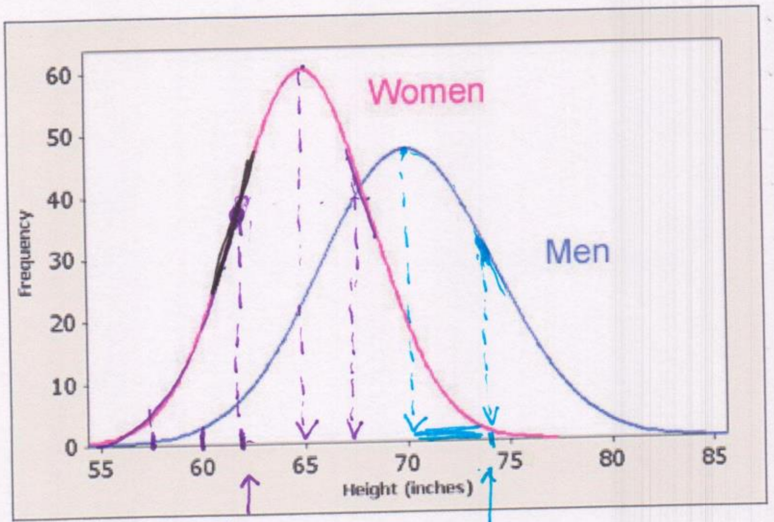
Distributions of Continuous Random Variables:
X = height is a continuous R.V. Suppose we had data on all the heights of all the men and women in the US.



Heights of people in the U.S., (assume entire population)

The distribution might not be perfectly normal, but it would be close enough that we could start saying some things about heights, as in what's an average height, what's pretty common and what's unusual.

Estimate the mean and standard deviation for each distribution. Comment on the spread and its relationship with the standard deviation. (Assume each curve represent the entire population and use proper notation.)



Women:
Mean $\mu_W = 65$ in
S.D. $\sigma_W = 2.5$ in (guess)

Men:
Mean $\mu_M = 70$ in
S.D $\sigma_{Men} = 4$ in

The S.D. is larger for men $\leftrightarrow$ more spread!

What are common heights for U.S. women, based on the information above?

Empirical Rule: "Common" within 1 S.D. of the mean

Women: Mean give or take 1 S.D

$\mu_W \pm 1\sigma_W$

65 in $\pm$ 1 (2.5 in)

60 in = 5 feet

Common heights | 62.5 in and 67.5 in

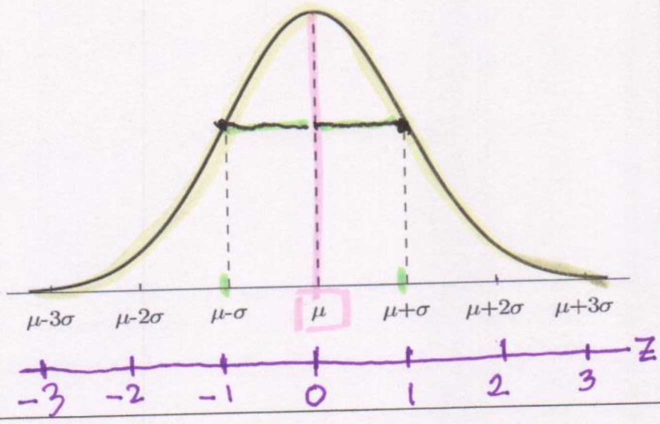
(−) (+)

If we know the mean and standard deviation we've see (in Section 3.2) that we can use z-scores and the Empirical Rule to judge what's common or unusual for a normally distributed Random Variable (like height).

The z-scores create their own normal curve, called the "Standard Normal Curve".

The Normal Distribution: The $N(\mu,\sigma)$ curve.

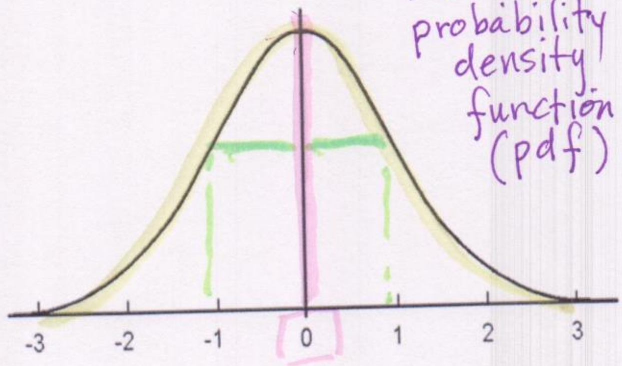
μ = population mean, σ = population standard deviation



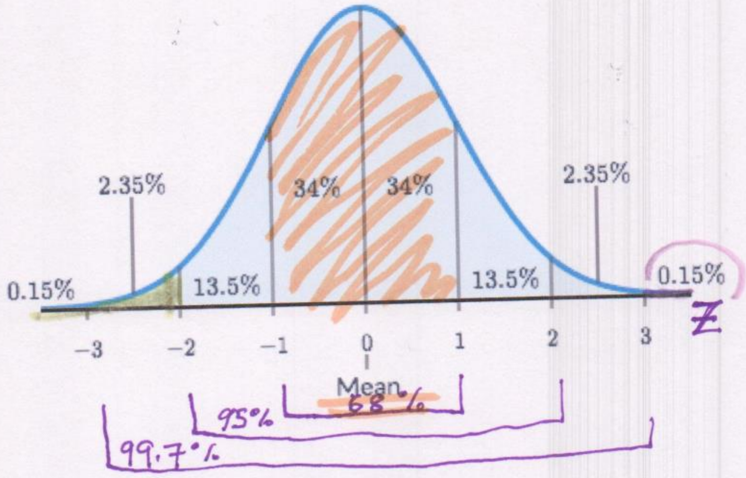
The Standard Normal Distribution: $N(0,1)$ curve

This curve is the "skeleton" underneath every normal distribution curve. $z = \dfrac{x-\mu}{\sigma}$

Gaussian Distribution probability density function (pdf)



The Empirical Rule, applied to the Standard Normal (z) Distribution looks like this:



Example: Write each problem using the proper notation. Then find the indicated probability using the Empirical Rule.

What's the probability of z being -2 or less? $P(z \le -2) = 2.35\% + .15\% = 2.5\% = .025$ $\qquad \frac{5\%}{2} = 2.5\%$

What's the probability of z being 3 or more? $P(z \ge 3) = .15\% = .0015$

What's the probability of z being between -1 and 1? $P(-1 \le z \le 1) = 68\% = .68$

What if our z-values aren't nice, perfect whole numbers? To find probabilities, we'll have to use technology.

Stop here and watch the video on how to use the StatCrunch Normal Distribution Calculator

Example: Write each problem using the proper notation. Then find the indicated probability using the StatCrunch Normal Distribution Calculator.

What's the probability of z being -1.73 or less? Standard $P(z \boxed{\leq} -1.73) = .04181514 = .042$

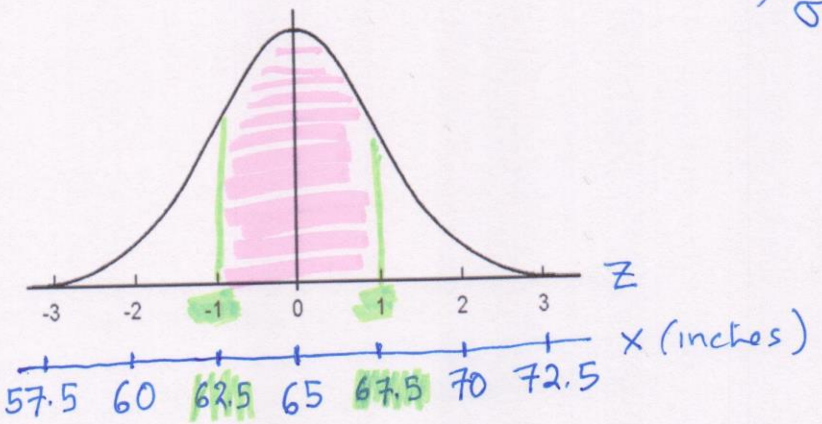
What's the probability of z being 2.18 or more? Standard $P(z \boxed{\geq} 2.18) = .01462873 = .015$

What's the probability of z being between -.93 and .93? Between $P(-.93 \leq z \leq .93) = .648$

Example: Heights of U.S. Women. The mean height for U.S. women is 65 inches, with a standard deviation of 2.5 inches.

Parking Lot
$\mu = 65$ in
$\sigma = 2.5$ in

Fill in the values of the heights on the z-score graph.



Find the percentage of women who are between 62.5 in and 67.5 inches
probability

(a) According to the Empirical Rule

$P(62.5 \leq x \leq 67.5) = 68\%$

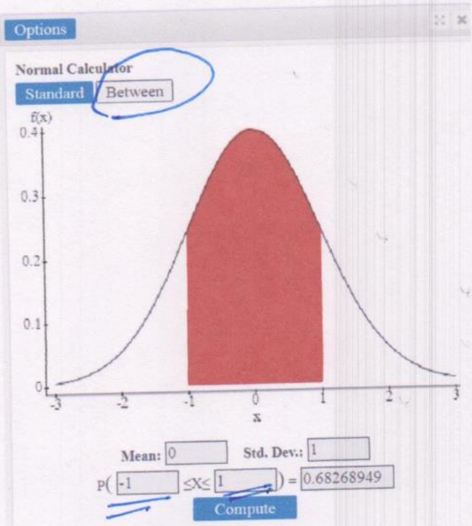
$\underbrace{P(-1 \leq z \leq 1)}_{\text{AREA}} = .68$

(b) Using the Normal Distribution calculator on StatCrunch:
Note: Use the z-scores!

$P(-1 \leq z \leq 1) =$

$= .68268949$

$= .683 = 68.3\%$ of women are between 62.5 in and 67.5 in in height



Example: Heights of U.S. women (continued)
Is a height of 5'9" = 69 inches unusual for a woman? (Mean height is 65 in, S.D. is 2.5 in)

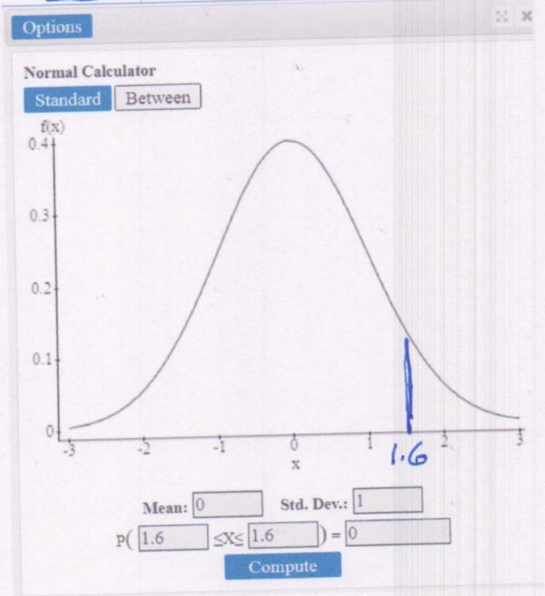
Parking Lot
$\mu = 65$ in
$\sigma = 2.5$ in
$x = 69$ in

Answer this question by finding the z-score for this height.

$$z = \frac{x-\mu}{\sigma} = \frac{69-65}{2.5} = 1.6$$

$z = 1.6$ SD's from mean (Not an unusual height)

Now, try find the probability that a woman is exactly 69 inches tall **using StatCrunch.**
Use the "Between" command and put the same z-score in both boxes.



$P(x = 69) = 0$

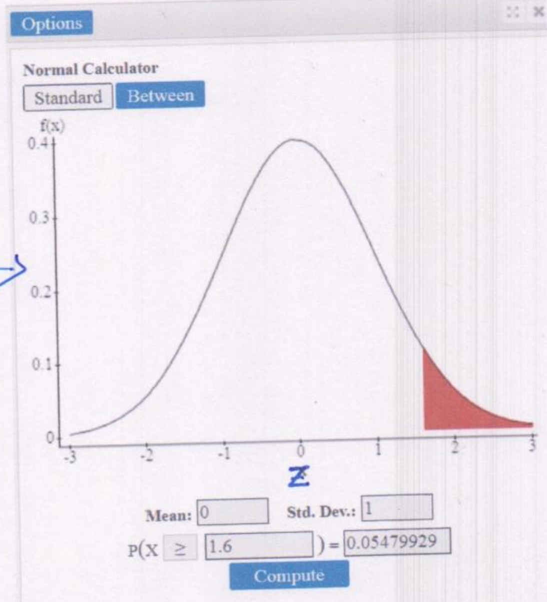
According to the model.

Model breakdown

Model only works in finding probabilities for ranges of values

Lines have no area! Areas - shaded

What is the probability a woman is 5'9" = 69 inches, or taller?



$P(x \geq 69)$

$= P(z \geq 1.6)$

$= .055$

$=$ 5.5% chance of a woman being 69 inches OR taller!

Finding probabilities using the Normal Distribution.

Example: For a certain type of computers, the length of time between charges of the battery is normally distributed with a mean of 50 hours and a standard deviation of 6 hours.

Parking Lot

$\mu = 50$ hrs

$\sigma = 6$ hrs

$x = 40$ hrs

Find the probability that the length of time between charges will be

x

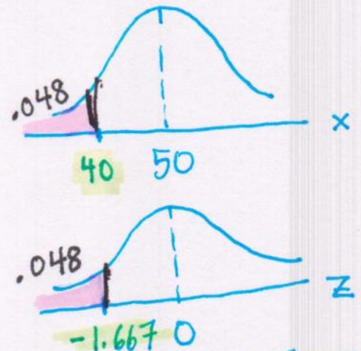
(a) less than 40 hours.

$P(x \le 40) = \underline{.048}$ same

$P(z \le -1.667) = \underline{.048}$ ← Use StatCrunch to answer this!

$$z = \frac{x - \mu}{\sigma}$$

$$= \frac{40 - 50}{6} = -1.667$$



(b) more than 62 hours.

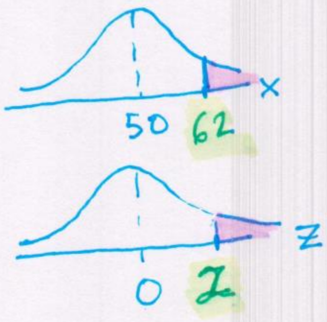
$P(x \ge 62) = \underline{.0227}$ same

$P(z \ge 2) = \underline{.0227}$

$\underline{.023}$ ~ more accurate!

$$z = \frac{62 - 50}{6} = \frac{12}{6} = 2$$

Emp. Rule: $P(z \ge 2) = 2.5\% = \underline{.025}$

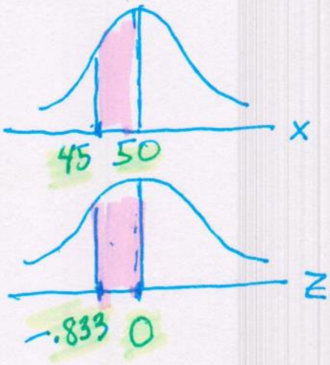


(c) between 45 and 50 hours

$P(45 \le x \le 50) = \underline{.298}$ ← same

$P(-.833 \le z \le 0) = \underline{.298}$

$x = 45$	$x = 50$
$z = \frac{45 - 50}{6}$	$z = \frac{50 - 50}{6}$
$= -\frac{10}{6} = -.833$	$= \frac{0}{6} = 0$

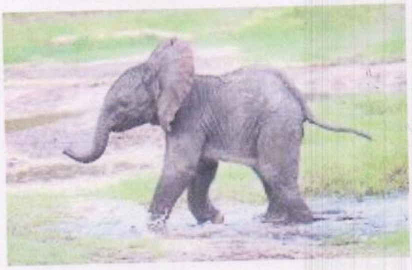


Data value → area = probability
x ← inverse — percentage

go backwards

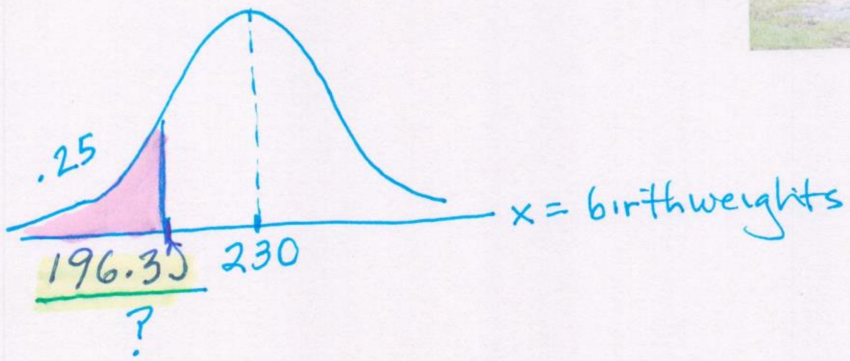
Inverse Normal Distribution: What if we know a percent (or probability) and want to find the x-value that goes with it? In order to that we have to use the Inverse Normal Distribution.

Percentile: The **kth percentile** of a data set is the x-value that has k **percent** (area) of the data **below** it.



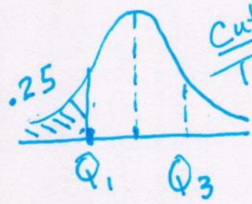
Example: The average (mean) birthweight of baby elephants is 230 pounds (wow!). Assume the distribution of birth weights follows the normal distribution N(230, 50). $N(\mu, \sigma)$

(a) Sketch a well-labeled curve for the distribution of baby elephant birth weights.



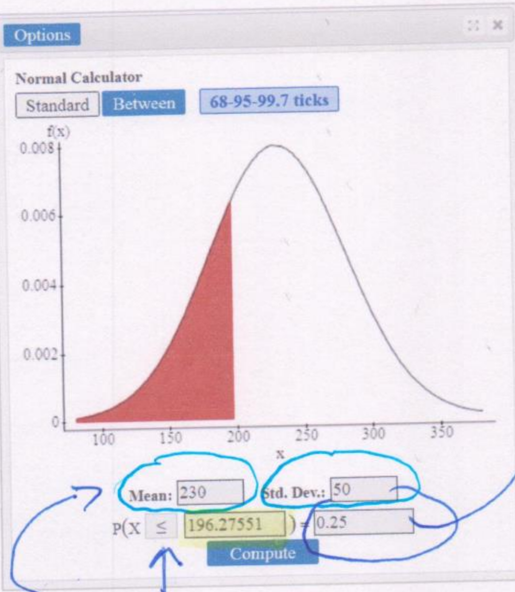
(b) Use the StatCrunch Normal Distribution Calculator to find the 25th percentile (x-value) for baby elephant birth weights.

Steps:
1. Sketch a Normal Curve with the indicated percentage shaded in. ✓
2. Pull up the Normal Distribution Calculator on StatCrunch
3. Fill in the mean and S.D.
3. Choose the ≤ symbol
4. Fill in the percentage in the probability box.
5. Click COMPUTE.



Cut off Top 25% same as bottom 75%

The 25th percentile = Q_1 for elephant birthweights is 196.3 pounds



Step 3

Fill in the desired percent

What is another name for the 25th percentile? Q_1